Parametric identification of the mathematical model of marine moving object using the apparatus of variational calculus

Sergey Pashentsev*, Uri Udin**, Sergey Kholichev***, Irina Glinskaya****

ABSTRACT

Quantitative discrepancy between the vessel motion parameters (when her propulsion system is used) obtained by means of simulation and those obtained in natural conditions raises the problem of the ship mathematical model efficiency when it is based on the results of model tests in experimental tanks (Hoffman A.D., 1988), in a wind tunnel and in automated control systems. Therefore, the problem of the parametric identification of the vessel mathematical model is very important (Moiseev, 1979), (Udin and Pashentsev, 2006). In this article the authors propose one of the approaches to solving this problem with the help of the mathematical apparatus of the variational calculus.

Keywords: marine mobile unit steering, the mathematical model of the vessel, parametric identification, variation calculus

* Cand.Sc. (Physics and Mathematics), professor, "Murmansk State Technical University", Marine Institute
** D.Sc., professor, "Murmansk State Technical University", Marine Institute
*** junior researcher, "Murmansk State Technical University", Marine Institute, corresponding author, tel: 7908-6077-255, E-mail: holichevsn@mstu.edu.ru
**** associated professor, "Murmansk State Technical University", Marine Institute

1. Introduction

The development of an adequate mathematical model of a marine mobile unit (MMU), for example, of a vessel – is an imperative condition for creating intelligent control systems of its movement in the water (Sobolev, 1976).

There is a common structure of the MMU mathematical model, which is made with regard to the known principles of the hydrodynamic theory, and there are a number of its varieties defined by features of the applied problems to be solved.

When a certain MMU mathematical model structure is chosen or formed on the basis of the objective of the applied problem to be solved, it is necessary to define model parameters.

On the first stage of solving an applied research problem or MMU steering problems its model parameters are defined with the help of results of numerous model experiments carried out in experimental tanks, rotative installations and wind tunnels. Taking into account the fact that experiments are carried out on models of one MMU type, but later as a rule these results are used to determine parameters of other MMU types, it is possible to assume low adequacy of the model developed this way. In this case the use of the mathematical model for solving an applied research problem or any other practical problem connected with the assessment of a MMU movement dynamics in real sea conditions becomes pointless.

Moreover, taking into account that the model experiment, for example, in the experimental tank, cannot fully cover all possible real variants of MMU state, the model parameters based on the results of the model experiment on the one hand and the parameters corresponding to the certain current state of a unit being modeled on the other hand will be markedly different.

Of course, it will reduce the quality of results of the applied model research conducted with the use of the MMU mathematical model as well as when using this model for solving the problems of this unit steerage.

2. The main principles of parametrical identification

For the above stated reasons it follows that the most important problem to be solved when planning, for example, model experiments both of research and applied type is the problem of parametrical identification of the MMU mathematical model, i.e. their values should be brought into line with the values corresponding to the current state of the given unit. It is particularly important if the MMU mathematical model is used in the adaptive control system, which is typical for modern intelligent control systems.

Sea trials, during which kinematic parameters of the MMU movement and the type of their changes are determined, should be considered the main source of data for its mathematical model parameters identification. Identification of the parameters of the mathematical model can take place continuously during MMU movement, not interrupting its principal activity, so that the obtained (identified) parameters of its model could immediately be used in the unit steering system and for predicting current and next MMU maneuvers. It is obvious that in this case the safety, the quality, and the efficiency of MMU steering will substantially increase.

Traditionally, the MMU mathematical model, for example, of a vessel is given in the form of differential equations of motion, and its parameters are the coefficients in the right-hand sides of these equations. Usually these factors enter into the right-side of equations linearly, although more complex variants of entering can be considered.

The problem of the parametric identification is usually defined as a problem of minimization of some functional in an integral form. If the set of unit state variables is designated by vector $X = \{X_i\}$, and the set of parameters of the MMU mathematical model is designated by vector $C = \{C_k\}$, the condition of the functional minimum will look as follows:

$$\min\{\int f(X, dX/dt, d^2X/dt^2, \overline{C}, t)dt\}, \ \overline{C} \in D \qquad (1)$$

where D is some closed variation area of model C parameters, and the integrand function can depend on both the state vector X and on its first dX/dt and second d^2X/dt^2 derivatives.

The concrete form of this function primarily depends on our ability to measure kinematic and dynamic parameters of MMU motion. Theoretically, when observing MMU motion we would like to measure 6 variables characterizing its movement – three linear accelerations $W = \{w_1, w_2, w_3\}$ and three angular accelerations $E = \{e_1, e_2, e_3\}$. By measuring these parameters we would be able to evaluate the kinematic characteristics of the six-dimensional motion: linear $V = (u_1, u_2, u_3)$ and angular speeds $W = \{w_1, w_2, w_3\}$. In this case it is essential that all these characteristics are determined by integration (- not by numerical differentiation of the measured variables) which greatly increases the accuracy of final results.

However, such a statement is only an idea, because the problem of fitting ordinary vessels with six-dimensional accelerometers and appropriate processing equipment hasn't been solved yet in required amount. That is why instead of the general problem (1) some particular problems of this type are to be solved depending on the dynamic and kinematic parameters of MMU motion which we can measure directly with the help of appropriate sensors. For example, if we measure kinematic parameters of MMU movement: speed V; x, y coordinates; and heading K; then the functional (1) can be represented in a form of a define integral within interval $(0, t_f)$:

$$\min\{\Phi = \int[a_1(X-X_э)^2+a_2(Y-Y_э)^2+a_3(V-V_э)^2+a_4(K-K_э)^2]dt\} = \min\{\Phi=\int F dt\}, \qquad (2)$$

where $X_э$, $Y_э$, $V_э$, $K_э$ are the values of kinematic parameters measured during MMU movement,

X, Y, V, K are the values of kinematic parameters of MMU movement which are determined in accordance with the chosen mathematical model, and therefore they are dependent on the parameters vector of the MMU mathematical model C,

$A = \{a_1, a_2, a_3, a_4\}$ - weight normalization vector, components of which define for us the significance of this or that kinematic parameter of MMU movement and reduce inhomogeneous summands of the integrand to a uniform dimension.

Problem (2) can easily be represented in a discrete form by replacing integral (2) by its discrete analog – the sum of integrand at points tk - the moments at which the kinematic parameters of motion linear speed Vkэ, drift angle bkэ and angular speed wkэ are measured. After that the problem can be solved quite traditionally by method of the least squares: the partial derivatives of the function being minimized with respect to the required parameters are set equal to zero. Now we have a so-called system of normal algebraic equations in accordance with the number of parameters being determined:

$$\partial\Phi/\partial Cj = 0 \quad j=1,\ 2,\dots,\ m \tag{3}$$

If the parameters themselves were included into the model linearly the obtained normal system is also linear and can be solved from a formal point of view without any apparent difficulties.

Despite the apparent simplicity of the problem stated this way its solution is confronted by a number of difficulties. Unobservability, i.e. impossibility to measure directly some kinematic parameters such as b, dw/dt, dV/dt makes it necessary to calculate them by means of differentiation one way or another. It significantly reduces accuracy of the final results. In addition, the matrix of the linear problem of the form (3) is ill-conditioned, and even small errors in the initial data lead to significant errors in determining the outcome parameters of the MMU mathematical model C (Udin et al, 2009), (Udin, Gololobov and Stepahno, 2009). Thus, two factors – the low accuracy of the initial information and the ill-conditioned matrix of the system (3) virtually put this problem into a class of ill-posed problems, the results of which are dangerous to trust. Therefore, getting back to the problem formulated by dependence (2) we will try to find acceptable approaches to solving it in other ways.

3. Variational approach

One of the approaches to solving problem (2) is the possibility of using the apparatus of variational calculus. Variational calculus formulates the necessary condition that function X is an extremal of the functional (2) in the form of the Euler-Lagrange equation (Elsgolts, 1969):

$$\frac{\partial F}{\partial X} - \frac{d}{dt}\left(\frac{\partial F}{\partial X'}\right) = 0 \tag{4}$$

As X is a multidimensional vector, then the partial derivatives are implied by derivatives with respect to this vector components, i.e. formally equation (4) transforms into a system of scalar differential equations with respect to a number of components of vector X. To obtain the solution of equation (4), we are to define boundary conditions at the ends of the integration interval:

$$X(0) = X0, \; X(tf) = Xf \tag{5}$$

Often due to the character of the problem the boundary condition on the right end of the integral tf is not defined and so it is replaced by so-called natural boundary condition:

$$F(tf) = 0, \; \frac{\partial F}{\partial X'}(tf) = 0$$

These two conditions are sufficient to solve the Euler-Lagrange equation – the second-order differential equation. Solving it we can find the parameters of the vessel mathematical model which are included in the solution found, and thereby we can complete the parametric identification of the model. Let us consider as an example two particular problems of the above stated procedure of identification.

Consider the example of two private tasks such identification procedure. This will be the simplest tasks with a small number of identified options to show the possibility of the proposed approach. The more complex issue of generalized model Nomoto with 6-s parameters fixed by us (Nomoto et al, 1957) and is in the stage of publishing.

4. Problem 1: Vessel acceleration

In this case the differential equations defining the linear motion of the vessel are of the form:

$$\begin{aligned} \frac{d\upsilon}{dt} &= C0 \times T_e - C1 \times \upsilon^2; \\ \frac{dx}{dt} &= \upsilon, \end{aligned} \tag{6}$$

where Te - the propulsive thrust.

Here we use a two-parameter mathematical model of the vessel, which contains the parameters C0 and C1, the values of which have an uncertainty, for example, due to errors in the value of the added mass of the ship and not only.

We will minimize the following functional:

$$\mathbf{min}\{\int[\alpha(X-X^{э})^2 + (V-V^{э})^2]dt\} = \mathbf{min}\{\int Fdt\}, \tag{7}$$

i.e. we will make the mathematical model (6), which describes the dynamics of vessel acceleration, be maximum adequate to longitudinal movement experimental data (in this case it doesn't matter which movement is considered) and to the linear speed of the vessel. Factor a = (1/tf)2 will be chosen as a weight factor, it makes the summands in equation (7) homogenous and equally important for us. Euler-Lagrange equation in this case is as follows:

$$\frac{\partial F}{\partial X}-\frac{d}{dt}(\frac{\partial F}{\partial X'})=\frac{\partial F}{\partial X}-\frac{d}{dt}(\frac{\partial F}{\partial V})=2\alpha(X-X^{э})-\frac{d}{dt}(2(V-V^{э}))=0,$$

as a result we have :

$$\alpha(x-x^{э})-\frac{d\upsilon}{dt}+\frac{d\upsilon^{э}}{dt}=0 \qquad (8)$$

Taking into consideration that dV/dt = d^2X/dt^2 we obtain the differential equation of the second order in the distance x:

$$\frac{d^2X}{dt^2}-\alpha X=-\alpha X^{э}+\frac{dV^{э}}{dt}=\varphi(t) \qquad (9)$$

It's a simple equation and can be written down as follows:

$$X(t)=D1(t)e^{t/tf}+D2(t)e^{-t/tf}$$

where coefficients D1(t) and D2(t) are determined by method of variation of constants in the form of integrals:

$$D1(t)=\int_0^{tf}\varphi(t)e^{-t/tf}dt \qquad D2(t)=\int_0^{tf}\varphi(t)e^{t/tf}dt \qquad (10)$$

However in our case we can obtain an algebraic equation in the vessel speed V.

For this purpose we should substitute the value of the speed derivative from equation (6) into equation (9), differentiate the obtained equation with respect to time, and again substitute the value of the speed derivative into it, i.e.:

$$\alpha(V-X'^{э})-C0*Pe'+2C1*V*(C0*Pe-C1*V^2)+\frac{dV^{э}}{dt}=0$$

As a result of these actions we will have an exponential equation of the third order in ship's speed V:

$$2C1^2V^3-V(\alpha+2C0*C1*Pe)+\alpha X'^{э}-V''^{э}+C0*Pe'=0 \qquad (11)$$

It is an incomplete third-order equation and its solution can be written down in the form of the Cardano formula:

$$V = \sqrt[3]{-q/2 + \sqrt{q^2/4 - p^3/27}} + \sqrt[3]{-q/2 - \sqrt{q^2/4 - p^3/27}}$$

where (12)

$$q = (\alpha + 2C0 * C1 * Pe)/2C1^2 \qquad p = (\alpha X'^{э} - V''^{э} + C0 * Pe')/2C1^2$$

Thus we get the problem solution, i.e. the equation of the extremals of functional (7) is found. However, our main goal is the mathematical model identification, that is, the definition of its parameters C0 and C1 in the first equation of system (6). Solution in the form (11) allows doing it by setting appropriate boundary conditions. At the initial time of movement the vessel speed and the travel have zero values: X(0)=0, V(0)=0.

At the final moment of the motion we can use the natural boundary condition, which can be represented by the equation u(tf) = ufэ.

Substituting the boundary values of variables x and u in equation (11), we obtain

$$C0 = \left[\left(\upsilon''^{э} - \alpha x'^{э} \right) / T_e' \right]_{t=0};$$

$$C1^2 * 2\upsilon_f^{э\,3} - C1 * 2C0T_{ef} + \alpha x_f'^{э} - \upsilon_f''^{э} + C0 * T_{ef}' - \upsilon_f^{э}\alpha = 0. \quad (13)$$

In the first equation (13) both variables representing the nature of the motion of the vessel, are taken at the initial time t = 0, in the second – at the final time t = tf, that is indexed by the subscript f. In the particular case, when at the final time the process becomes steady, the second equation (13) is greatly simplified by zero derivatives of the variables representing the character of motion, which leads to more simple equation

$$C1 = C0 \times T_{ef} / \upsilon_f^2 .$$

5. Numerical solution

Let's consider a numerical example of how to use this method for solving a specific practical problem. The results of solution of the vessel movement problem will be taken as conditional experimental data in accordance with the dependence:

$$\frac{d\upsilon}{dt} = B0 \times T_e - B1 \times \upsilon^{2,1}$$

where (6) C0= 0.3, C1 = 0.58, and the power of speed in the conventional law describing the hydrodynamic resistance to the ship movement is deliberately taken to be equal to 2,1. In this case, it is assumed that propulsive thrust Te increases linearly, and then remains constant. We obtain the problem solution in MathCad system, its fragments are given below as Figure 1, Figure 2, Figure 3.

Figure 1. The example of obtaining experimental data by means of solving a problem with the given parameters (acceleration).

$$C0 := 0.3 \qquad C1 := 0.58 \qquad Pe(t) := \begin{vmatrix} 5\frac{t}{3} & \text{if } t < 3 \\ 5 & \text{if } t \geq 3 \end{vmatrix} \qquad x := \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$D(t,x) := \begin{bmatrix} C0 \cdot Pe(t) - C1 \cdot \left(x^{2.1}\right)_0 \\ x_0 \end{bmatrix}$$

$$n := 100 \qquad m := 10 \qquad k := 0..n$$

$$Z := \mathrm{rkfixed}(x, 0, m, n, D)$$

$$V := Z^{\langle 1 \rangle} \qquad X := Z^{\langle 2 \rangle} \qquad T := Z^{\langle 0 \rangle}$$

Thus we determine the parameters being identified by means of formulas (13). The derivatives of the variables characterizing the movement of the vessel are found using finite differences, as shown in one of the fragments of Figure 2. This solution gives the identified values of the model parameters:

Figure 2. The identification of model parameters by means of formula (13).

$$dt := \frac{m}{n}$$

$$Pes0 := \frac{Pe(dt) - Pe(0)}{dt} \qquad Xs0 := \frac{X_1 - X_0}{dt} \qquad Vs0 := \frac{V_1 - V_0}{dt}$$

$$Vss0 := \frac{(-2V)_1 + V_2 + V_0}{dt^2} \qquad Vssf := \frac{(-2V)_{n-1} + V_{n-2} + V_n}{dt^2} \qquad Xsf := \frac{X_{n-1} - X_n}{-dt}$$

$$CC0 := \frac{Vss0 - Xs0}{Pes0}$$

$$Pef := Pe(m) \qquad Vf := V_n$$

$$CC1 := \frac{CC0 \cdot Pef}{Vf^2}$$

C0 = 0.299, C1 = 0.606. This is a very good result, because the actual values were 0.3 and 0.58. The first value is almost exact, the second has 4.5 % error.

Then we solve the problem which directly describes our unit movement with the help of a model, the parameters in which have the identified values B0 = 0.3, B0 = 0.606, and the resistance is proportional to the second power of speed in accordance with the conventional hydrodynamic theory postulates.

The aim of this solution is to calculate the value of the functional which appears to be equal to J = 0.837. The relevant fragment of the solution is given in Fig.3

Figure 3. The calculation of the functional with the identified parameters (J = 0.837).

$$y := \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad DD(t,y) := \begin{bmatrix} B0 \cdot Pe(t) - B1 \cdot \left(y^2\right)_0 \\ y_0 \end{bmatrix}$$

ZZ := rkfixed(x, 0, m, n, DD)

$$VV := ZZ^{\langle 1 \rangle} \qquad XX := ZZ^{\langle 2 \rangle}$$

The calculation of the functional

$$J := \sum_{k=0}^{n} \left[\left(VV_k - V_k\right)^2 + \frac{\left(XX_k - X_k\right)^2}{tf^2} \right]$$

6. Problem 2: vessel circulation

In this case the differential equations which govern the curvilinear motion have the following form:

$$\begin{aligned} d\omega/dt &= -\omega/\tau_1 + K^{\delta}_{\omega}\delta(t) \\ dK/dt &= \delta(t), \end{aligned} \qquad (14)$$

where d(t) is the rudder angle , K - the ship's speed, w - the angular speed of the vessel.

The first equation of this system is called Nomoto equation (Sobolev, 1976), (Elsgolts, 1969) and is widely used for studying the yawing process.

Similarly to the previous example here we also use the two-parameter mathematical model of the vessel containing the parameters t1 and Kdw, which are called the main inertia constant and the variable characterizing the vessel's initial turning qualities (Sobolev, 1976).

Let's minimize the following functional:

$$\mathbf{min}\{\int[\alpha(K-K^{э})^2 + (\omega-\omega^{э})^2]dt\} = \mathbf{min}\{\int F dt\}, \qquad (15)$$

i.e. again we want the mathematical model to be maximum adequate to experimental data on the heading K and angular speed w. The known to us multiplier a = (1/tf)2 will be chosen as a weight factor. It makes the summands of the equation (15) homogenous and equivalent. Euler-Lagrange equation in this case is as follows:

$$\frac{\partial F}{\partial X}-\frac{d}{dt}(\frac{\partial F}{\partial X'})=\frac{\partial F}{\partial K}-\frac{d}{dt}(\frac{\partial F}{\partial \omega})=2\alpha(K-X^{э})-\frac{d}{dt}(2(\omega-\omega^{э}))=0,$$

finally it will look like this:

$$\alpha(K-K^{э})-\frac{d\omega}{dt}+\frac{d\omega^{э}}{dt}=0 \qquad (16)$$

Taking into account that dw/dt = d2K/dt2 we obtain the differential equation of the second order for the extremal with respect to the vessel heading K:

$$\frac{d^2K}{dt^2}-\alpha K=\alpha K^{э}+\frac{d\omega^{э}}{dt}=\psi(t) \qquad (17)$$

Principle of solving equations of this type is given above (10), the general form of the solution is given in the form of the dependence of the following type:

$$K(t)=E1(t)\times e^{t/t_f}+E2(t)\times e^{-t/t_f},$$

where coefficients E1(t) and E2(t) are determined by the method of variation of constants in integral forms:

$$E1(t)=\int_0^{tf}\psi(t)e^{-t/tf}dt \qquad E2(t)=\int_0^{tf}\psi(t)e^{t/tf}dt \qquad (18)$$

However in our case we can obtain algebraic equation in the vessel angular speed w. Let's substitute the value of the derivative of angular speed from equation (14) in equation (16), differentiate the obtained equation with respect to time and again substitute the value of the derivative of the angular speed into it. We will obtain the linear equation in w:

$$\omega(\alpha-1/\tau_1^2)=\alpha K'^{э}-K_\omega^\delta\delta/\tau_1+K_\omega^\delta\delta'-\omega''^{э},$$

from which we can easily find the angular speed

$$\omega=(\alpha K'^{э}-K_\omega^\delta\delta/\tau_1+K_\omega^\delta\delta'-\omega''^{э})/(\alpha-1/\tau_1^2) \qquad (19)$$

Now it is possible to identify the parameters of the mathematical model represented by the system of equations (14), using the initial condition at the left end of the interval w(0) = 0 and the natural boundary condition at the right end w(tf) = wfэ. Taking into consideration these conditions we will obtain a system of two algebraic equations from which we can find the parameters:

$$K_{\omega}^{\delta} = (\omega''^{\text{э}}(0) - K'^{\text{э}}(0)/t_f^2)/\delta'(0)$$
$$\omega_f/\tau^2{}_1 - K_{\omega}^{\delta}\delta_f/\tau_1 + (K_f'^{\text{э}}/t_f^2 - \omega_f/t_f^2 + K_{\omega}^{\delta}\delta_f' - \omega_f''^{\text{э}}) = 0 \qquad (20)$$

If we assume the stationarity of the final state of the vessel, then from the second equation (20) we can find the second parameter being identified

$$1/\tau_1 = K_{\omega}^{\delta}\delta_f/\omega_f \qquad (21)$$

7. Numerical solution

Let's consider a computational example of using this approach in the same way it has been done for the problem of vessel acceleration. As experimental data we take the results of solving the problem of the ship's initial state during circulation with the change of angular speed by law:

$$d\omega/dt = -B0\ \omega^{1.1} + B1\ \delta(t),$$

in which the parameters C0= 1.6, C1 = 0.56, and the power in the law of resistance is deliberately chosen not equal to1.0 but to1.1 for experimental data distortion. Rudder angle increases linearly up to 0.5 rad, then remains constant. Fragments of calculation in MathCad are shown on Figure 4, Figure 5, Figure 6.

Figure 4. The way of obtaining experimental data by solving the problem with the given parameters (circulation).

$C0 := 1.6 \qquad C1 := 0.56 \qquad DelR(t) := \begin{cases} .5\dfrac{t}{60} & \text{if } t < 60 \\ 0.5 & \text{if } t \ge 60 \end{cases}$

$$D(t,x) := \begin{bmatrix} C0 \cdot DelR(t) - C1 \cdot \left(x^{1.1}\right)_0 \\ x_0 \end{bmatrix} \qquad x := \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$n := 1000 \qquad m := 100$

$Z := \text{rkfixed}(x, 0, m, n, D) \qquad k := 0..\ n - 2$

$OM := Z^{\langle 1 \rangle} \qquad K := Z^{\langle 2 \rangle} \qquad T := Z^{\langle 0 \rangle}$

Directly identifiable parameters are obtained with the help of formulas from groups (20), (21). Besides that, the derivatives of movement characteristics are obtained by means of finite differences as it is shown in Fig.5. This solution gives the identified values of the model parameters: C0 = 1.562, C1 = 0.565. Again the result is good, because the actual values were 1.6 and 0.565. Both values have the error of not more than 2%.

Figure 5. Identification of model parameters by the formulas (19), (20).

$$tf := m$$

$$DelRs0 := \frac{DelR(dt) - DelR(0)}{dt} \qquad Ks0 := \frac{K_1 - K_0}{dt}$$

$$OMs0 := \frac{OM_1 - OM_0}{dt} \qquad OMss0 := \frac{(-2OM)_1 + OM_2 + OM_0}{dt^2} \qquad OMf := OM_n$$

$$CC0 := \frac{OMss0 - \frac{Ks0}{tf^2}}{DelRs0}$$

$$Ksf := \frac{K_{n-1} - K_n}{-dt} \qquad OMssf := \frac{(-2OM)_{n-1} + OM_{n-2} + OM_n}{dt^2}$$

$$DelRsf := \frac{DelR(tf - dt) - DelR(tf)}{dt} \qquad DelRf := DelR(tf)$$

$$CC1 := \frac{(CC0 \cdot DelRf)}{OMf}$$

Now let's solve the problem which describes the circulation of a unit with the help of a model, in which the parameters have the identified values B0 = 1.562, B1 = 0.565, and the damping is proportional to the first power of the angular speed. The aim of solving this problem is to calculate the value of the functional which is equal to J = 1.073. The appropriate fragment of this calculation is shown in Fig 6.

Figure 6. The calculation of the functional with the identified parameters (J = 1.073).

$$B0 := 1.6 \qquad B1 := 0.56 \qquad y := \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$DD(t, y) := \begin{bmatrix} B0 \cdot DelR(t) - B1 \cdot (y)_0 \\ y_0 \end{bmatrix}$$

$$ZZ := rkfixed(x, 0, m, n, DD)$$

$$OMM := ZZ^{\langle 1 \rangle} \qquad KK := ZZ^{\langle 2 \rangle}$$

The calculation of the functional

$$J := \sum_{k=0}^{n} \left[(OMM_k - OM_k)^2 + \frac{(KK_k - K_k)^2}{tf^2} \right]$$

8. Conclusion

In this article we proposed and described variational approach to solving problems of parametric identification of ship mathematical models for different purposes. On the basis of the article materials we can make a conclusion that for models with a small number of parameters (mathematical models with few parameters), this approach provides sufficiently accurate results and can be used to approximate compound motions by a set of simple motions, models of which are easily identified. Its application to models with a large number of parameters requires further research, which is being carried out by the authors of this article.

Reference

Elsgolts L.E. (1969) Differential equations and the calculus of variations, M.: Nauka, 375 p.

Hoffman A.D. (1988) Propulsion and maneuverability, Directory, Leningrad Shipbuilding, 360 p.

Moiseev N.N. (1979) Numerical methods for the synthesis of optimal controls, M.: Nauka, 443 p.

Nomoto, Taguchi, Honda, Hirano (1957) On Steering Qualities of Ships, -JSP. 35:56-64.

Sobolev G.V. (1976) Ship handling and automation of navigation, L.: Shipbuilding, 478 p.

Udin U.I., Gololobov A.N., Barakhta A.V., Stepahno A.G. (2009) The problem of ensuring safe navigation during oil products logistics in the Russian Arctic shelf, Murmansk: Vestnik MSTU. 12(1):13-16.

Udin U.I., Gololobov A.N., Stepahno A.G. (2009) Identified vessel's mathematical models application for safe navigation, Vestnk Bauman. 12(1):10-12.

Udin U.I., Pashentsev S.V. (2006) Using the Pontryagin maximum principle for parametric identification of a mathematical model of the vessel, Science and Technology of Transport. 2:100-107.